Why Gravity is True

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Flat Earth proponents do not believe that there is such a thing as gravity. In this document I'll address the experimental and observational evidence that support Newton's theory of gravity, and the history of how he arrived at the theory. (Please note that Newton's theory has since been superseded by Einstein's theory of General Relativity, but Newton's theory remains a very accurate approximation to General Relativity except under certain extreme conditions.)

1 Newton's Laws of Motion

1.1 Vectors and Scalars

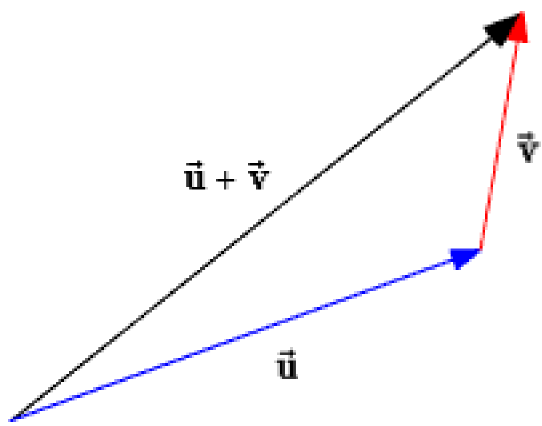
To understand Newton's Laws of Motion, let's first talk about vectors vs. scalars:

- A scalar is just a simple quantity.
  - Weight is a scalar.
  - Duration of time is a scalar.
  - Speed is a scalar.
  - Mass is a scalar.
- A vector is a *directed* quantity. It has a direction and a magnitude. Think of it as an arrow pointing in a certain direction, with the magnitude being the length of the arrow.
  - Location is a vector: it is the distance from a chosen point of reference, plus the direction from the reference point to the given location.
  - Velocity is a vector. Velocity is a speed *plus* the direction of motion.
  - Acceleration is a vector: it is the rate of change of velocity. Note that *deceleration* is just acceleration in the opposite direction. For example, when you hit the brakes on your car, you accelerate backwards.
  - Force is a vector: it has a strength (measured in pounds or Newtons) and a direction. A force pushing you downward is different than a force of the same magnitude pushing you to the side.

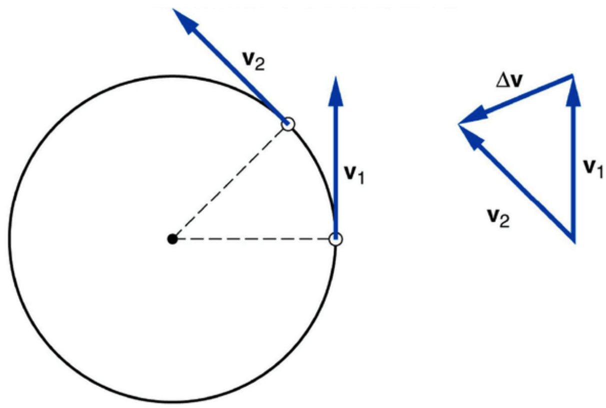
You can do certain kinds of arithmetic with vectors:

- You can multiply a vector by a scalar. Multiplying by a scalar c is the same as multiplying the magnitude by c while keeping the direction the same. For example, multiplying the velocity {30 mph horizontally due north} by 2, you get another vector, {60 mph horizontally due north}. If you multiply a vector by -1, you get a vector with the same magnitude, pointing in the opposite direction.

- You can add two vectors. You do this by taking one of the arrows and sliding its origin to the end of the other vector (without changing its direction), as shown here:



Since velocity is a vector, any change in direction, even if the speed stays the same, is an acceleration. In particular, suppose you have an object undergoing circular motion with a constant speed. Then the object is accelerating, and the direction of acceleration is inwards, towards the center of the circle. The following diagram shows why:



In the above, v_1 is the velocity vector when the circling object is at the tail of the arrow, and v_2 is its velocity vector after traveling a short distance along the circle. The acceleration is the change of velocity Δv (shown to the right) divided by the elapsed time. You can see that the vector Δv points inward to the center of the circle.

1.2 The Laws of Motion

These are Newton's Three Laws of Motion:

Every object in a state of uniform motion will remain in that state of motion unless an external force acts on it.

In other words, *there can be no acceleration (change in velocity) without some external force.*

You might question this law, as we're used to thrown or pushed objects eventually coming to a rest. In this case friction provides the decelerating force: air friction for a thrown ball, water friction for a boat on the water, surface friction for a hockey puck sliding along the ice. When you slide an object along ice, the slipperier the ice (lower friction) the longer the object continues moving before coming to a rest. A plastic puck on an air hockey table continues moving longer when the air is on than when the air is off, because air friction is considerably less than the table surface friction.

Force equals mass times acceleration

That is, $f(t) = m \cdot a(t)$, where m is the mass of an object, $a(t)$ is the acceleration vector at time t, and $f(t)$ is the net force on an object at time t. *Net force* is the sum of all the forces acting on the object, using the vector addition described earlier.

For every action there is an equal and opposite reaction

That is, if object A exerts a force f on object B, then object B exerts a force $-f$ on object A. This ensures that the forces acting *within* an object or a collection of objects give rise to no net acceleration. You can't move your boat forward by pushing on it while standing on the deck.

1.3 Why Do We Believe These Laws?

The short answer is that they've been verified by countless laboratory experiments over the last several hundred years, and that mechanical engineering, structural engineering, etc. are founded on these laws. If they were wrong, our vehicles would not work, industrial robots would not work, buildings would not stay up, etc. (Structural engineering, which focuses on static structures, depends on these laws because the net force has to be zero for things stay put.) These laws are tested millions of times every day by the operation of every car, boat, plane, industrial robot, fan, turbine, etc. and by every building, highway overpass, dam, etc.

2 Deriving Gravity

2.1 Newton's Law of Gravity

Newton's Law of Gravity is as follows: any two objects A and B have a gravitational force of magnitude

$$f = \frac{Gm_A m_B}{r^2}$$

acting between them, where

- m_A is the mass of object A;
- m_B is the mass of object B;
- r is the distance between the centers of mass of the two objects; and
- G is some constant of proportionality (the gravitational constant.)

That is,

- object A exerts a gravitational force of magnitude f, pointing towards A, on object B; and
- object B exerts a gravitational force of the same magnitude, pointing towards B, on object A.

Where did this come from? How did Newton come up with this formula?

- Unsupported objects fall towards the Earth with a constant acceleration. Therefore some force is acting on the falling object. "Gravity" is as good a name for this force as any.

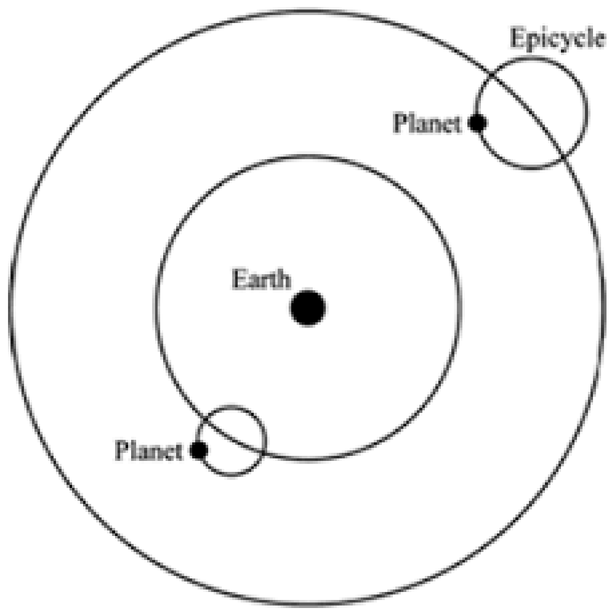
- Since the acceleration (and hence the force) points towards the Earth, the most obvious hypothesis is that the Earth itself is exerting the force.
- Furthermore, all objects on the Earth fall with the same acceleration, except for differences caused by air friction.
- From $f = m \cdot a$, and a is the same for all objects, we conclude that the gravitational force of the Earth on an object must be proportional to the mass of the object.
- From the Third Law of Motion, the object must exert an equal and opposite force on the Earth; one would then expect that *its* force on the Earth would likewise be proportional to the Earth's mass, by exchanging the roles of Earth and object.
- This gives us the Gm_Am_B part, but what about the division by r^2? You might suspect that the gravitational force weakens with distance—otherwise any object, no matter how far away in the cosmos, could be exerting strong gravitational forces on us—but all objects on the surface of the Earth are nearly the same distance from its center, making it difficult to experimentally investigate the dependence on distance.

The answer to this last question—how gravity depends on distance—came from observing the planets.

2.2 Kepler's Laws

The word "planet" comes from the Greek phrases *aster planetes*, meaning "wandering stars." Whereas the "fixed stars" appear to remain stationary with respect to each other (albeit appearing to rotate around the Earth about once a day), the planets appear to move with respect to this backdrop of fixed stars. The ancients included the Sun and Moon among the planets, because they too appear to move relative to the fixed stars.

Ancient astronomers were interested in predicting the motions of the planets, and around AD 150 the astronomer Ptolemy devised a system based on "epicycles." He had a spherical Earth (yes, *spherical*) in the center of the cosmos, with the planets (including Sun and Moon) circling it in uniform circular motion… almost. Actually, he had each planet circling a point that itself was circling the Earth; these are the epicycles. (400 years earlier Aristarchus of Samos had proposed a heliocentric model, based on calculations that showed the Sun to be much larger than the Earth, but his model never got much traction.)

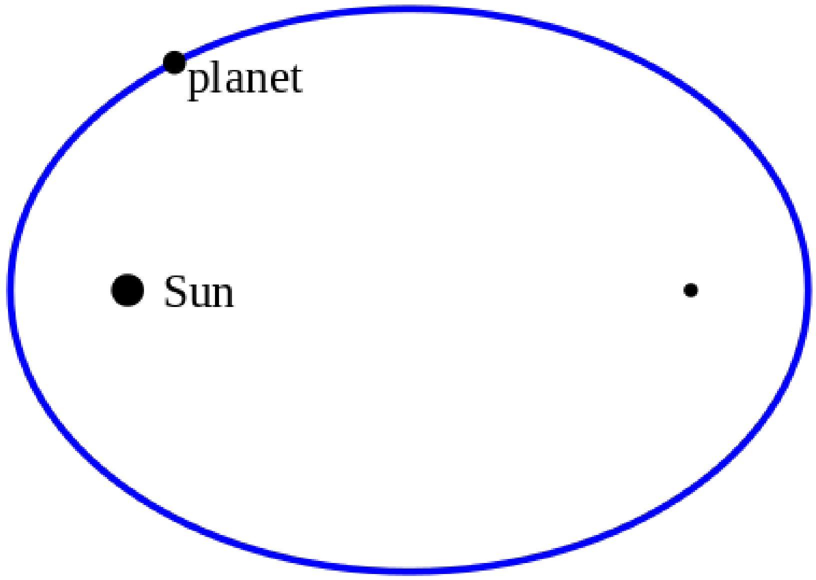


This Ptolemaic system was used for 1500 years, and it sort of worked, but over time astronomers became increasingly dissatisfied with its inaccuracies. In 1543 Copernicus proposed a heliocentric model that also used perfect circles and epicycles; it was a bit simpler, but not much more accurate. The big breakthrough came with Kepler in 1621.

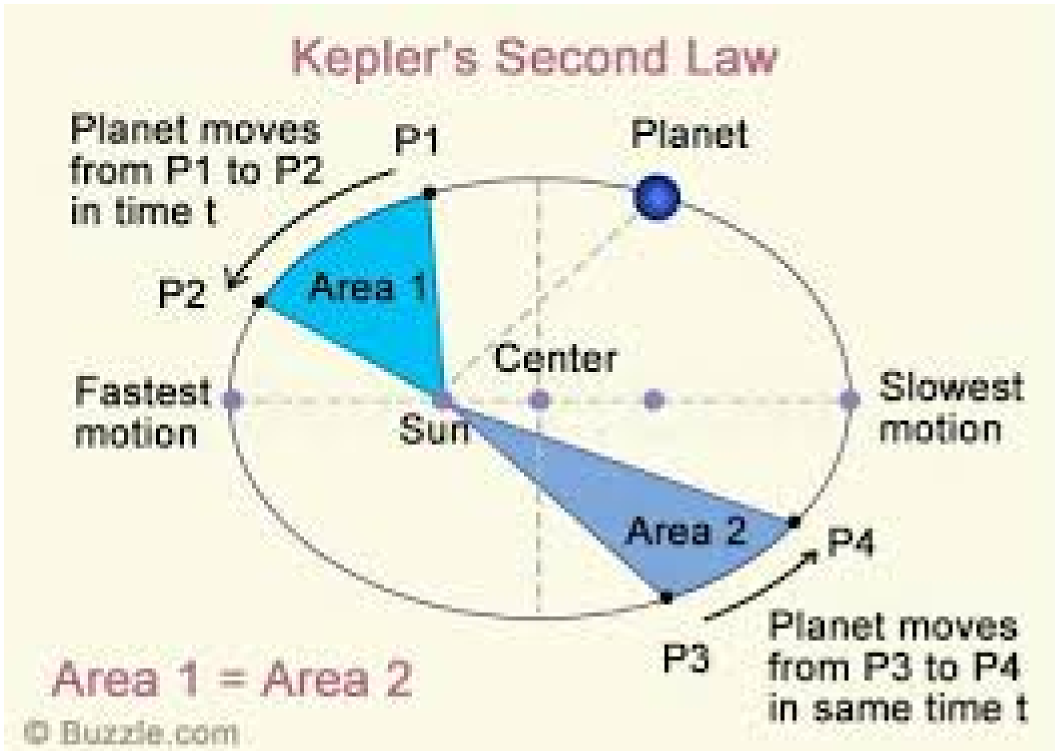
Tycho Brahe, known as the greatest of the naked-eye astronomers (he died just before the telescope was invented), carried out a very meticulous program of astronomical observation over a period of decades. He built the most accurate astronomical instruments of his day, and so compiled both a quantity and quality of planetary observations beyond what anyone else had heretofore achieved.

His assistant Johannes Kepler analyzed this data and from it deduced *Kepler's laws of planetary motion*:

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.



2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. (This implies that the planet travels faster when it is closer to the Sun.)



3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

In Kepler's system the planets and Earth orbited the Sun, and the Moon orbited the Earth. His first two laws could also be applied to the Moon's motion; the third law was moot, as there was no other object orbiting the Earth to which the Moon's orbit could be compared.

Why believe Kepler's heliocentric model and his three laws? *Because they gave the right answers.* They predicted the apparent motions of the Moon, Sun, and planets much more accurately than did the Ptolemaic system.

Note that Flat Earth theories are even worse than the Ptolemaic system: they give you *no means whatsoever* to predict the apparent motions of the Sun, Moon, and planets.

2.3 Using Kepler's Laws

In 1659 Huygens analyzed constant circular motion, and found that it implied a constant inwards acceleration of magnitude v^2/r, where v is the linear speed and r is the radius of the circle. But what about the planetary motions in Kepler's model, with their varying speeds and elliptical orbits? What sort of acceleration did they imply?

Newton invented calculus specifically to solve problems like this. Kepler's laws tell you how fast a planet moves about its orbit, as a function of time, up to some constant factor. This gives you a function $x(t)$ for the planet's position. Using Newton's calculus one can compute the planet's velocity $v(t)$ by taking the *derivative* of $x(t)$. And one obtains its acceleration $a(t)$ as the derivative of $v(t)$. When Newton did this, he found something very interesting: a planet's acceleration is always directed towards the Sun, just as you would expect if this acceleration were caused by a gravitational force from the Sun. Furthermore, this acceleration (and hence the force) is proportional to $1/r^2$. This provided the final piece of the gravitational force puzzle : divide by r^2 to get the dependence of the gravitational force on distance.

Let's do a quick rough check to see if this is correct, by applying it to the Moon's orbit. At the surface of the Earth, gravity produces an acceleration of 32.2 feet per second, per second. Calculations dating back to Eratosthenes in 300 BC give the earth a diameter of about 4000 miles (3959 miles, to be precise). Observations and calculations using parallax, dating back to Aristarchus in 270 BC, give a distance to the moon of about 240,000 miles (238,900 miles, to be precise.) That means the Moon is 60.34 Earth radii distant, and so the gravitational acceleration on it from the Earth should be

$$32.2 \text{ ft/s}^2/60.34^2 = 8.84 \times 10^{-3} \text{ ft/s}^2.$$

Approximating it as a circle, the length of the Moon's orbit is approximately

$$2\pi \cdot 238900\,\text{miles} = 1.501\,\text{million miles} = 7.93 \times 10^9\,\text{feet}$$

and it takes 27.322 days to complete an orbit, so its orbital speed is about

$$7.93 \times 10^9/27.322/24/60/60 = 3360\,\text{ft/s}.$$

Using Huygens' formula, to stay in a circular orbit of radius r at a speed v requires an acceleration towards the center of the circle of v^2/r. Using an Earth-Moon distance of 238,900 miles, which is 1.261×10^9 feet, this works out to

$$\frac{v^2}{r} = \frac{3360^2}{1.261 \times 10^9} = 8.953 \times 10^{-3}\text{ft/s}^2$$

which is a reasonably close match to the computed gravitational acceleration. The small discrepancy arises because the Moon's orbit is actually mildly elliptical, and we approximated it as circular.

3 Why Believe Newton's Law of Gravity

The above derivation makes a pretty strong plausibility case, but we can do better. Here are several strong reasons for believing that Newton's Law of Gravity is, at the least, a very good approximation to the truth:

1. *Newtons Laws get the right answers in predicting the motion of the planets.* Kepler's Laws are much more accurate than the Ptolemaic systems, but they aren't perfectly accurate. It turns out that you can mathematically derive Kepler's Laws from Newton's Law of Gravity and Laws of Motion... if you ignore the gravitational forces between the planets, which are much smaller than the gravitational force between the Sun and each individual planet. But if you calculate the motion directly using Newton's Laws, including the gravitational forces between all the planets, the results are very accurate, more so than Kepler's Laws.

   Furthermore, the previously unknown planet Neptune was *predicted to exist* based on Newton's laws. There were discrepancies in the orbit of Uranus that could be explained if another large planet existed beyond Uranus. French astronomer Urbain Le Verrier calculated where this planet would have to be, another astronomer looked where Verrier's calculations said to look, and there he found Neptune.

   Likewise, we get very accurate predictions for the orbits of the Moon and asteroids and moons of other planets using Newton's Laws. Both lunar and solar eclipses are accurately predicted *in advance* using Newton's Laws. Not only the time of the eclipse, but whether there is a total eclipse, partial eclipse, or annular eclipse; and for a total solar eclipse, what locations see a total (solar) eclipse, no eclipse, or a partial eclipse (and how partial).

   Today there remains only *one* discrepancy between a planetary orbit and the predictions of Newton's Laws: the precession of Mercury. The major axis of the elliptical orbit of Mercury very slightly changes direction over many years. It's a very subtle effect, and some, but not all, of this precession can be accounted for using Newton's theories. The very small discrepancy was finally resolved when Einstein developed his General Theory of Relativity, a theory of gravity that replaced Newton's and correctly predicts the precession of the orbit of Mercury, along with several other unexpected phenomena. Newton's theory, it turns out, is a very good approximation to Einstein's newer theory in almost all cases.

   Need I mention that Flat Earth theories are not even in the running here? They have no explanation whatsoever for the motions of the planets; they can't predict eclipses; they can't even predict what the Ptolemaic system could 2000 years ago.

2. *You can actually measure the gravitational force between two objects in the laboratory.* British scientist Henry Cavendish was the first to do this, over 200 years ago (1797–1798). Using a very sensitive apparatus, he was able to measure the force of gravitational attraction between a 12-inch lead sphere and a 2-inch lead sphere. You can buy the experimental apparatus to do this yourself from PASCO; the Harvard Natural Sciences Lecture Demonstrations catalog also gives instructions for carrying out the experiment.

3. *You have to recalibrate scales when you change elevation or latitude.* Newton's Law of Gravity says that the force of gravity decreases as you get farther from the center of the Earth: hence, slightly lower gravity at higher elevations. His Laws of Motion predict that the rotation of the Earth should produce an apparent outward "centrifugal" force that partially cancels the pull of of gravity, increasing from nothing at the poles to a maximum at the equator. Hence, lower apparent gravity as you go from poles to equator. The centrifugal force also produces an equatorial bulge in the shape of the Earth, so that it's slightly flattened, meaning that sea level at the equator is farther from the center of the Earth than sea level at the poles. This also reduces the actual gravitational pull as you go from the poles to the equator.

   All of this has practical, measurable consequences. Since there is a variation of up to half a percent in the force of gravity depending on where you are on the Earth, scales and other instruments that rely on the force of gravity have to be recalibrated when you take them to a different location at a different elevation and/or latitude. Here are a couple of web pages where companies explain this to their customers: one from the instrument manufacturer Shimadzu, and another from the online publication Digital Engineering.

   Flat Earth theories, which deny gravity, predict neither the existence of this effect nor its magnitude. Newton's Laws of Gravity and Motion both predict the effect and correctly predict the magnitude.

4. *Newtons Laws predict the timing and magnitude of the tides.* Newton's Laws predict the existence of tidal effects when gravitational forces act on any extended body. This is a simple consequence of the gravitational force being proportional to $1/r^2$—the farther side of a body experiences less gravitational force than the nearer side. The net result is a stretching force on the body, in the direction of the source of gravitational pull. The Moon's tidal effect on the earth creates two bulges of water in the oceans, which is why you get two tides a day.

   The Sun also produces tides, which is one reason why tides get complicated. But since the tidal force is the *difference* in gravitational force from one side of the Earth to the other, and the Moon is much closer, the Sun's tidal effect is smaller than the Moon's, even though its gravitational force on the Earth is stronger.

   Bottom line is that scientists can predict the magnitude and timing of the tides using Newton's laws, information about the Sun and Moon, and other factors. Flat Earthers cannot.

5. *Mining companies use gravitational measurements to detect mineral deposits.* There is something called a *gravity survey*, which measures minute changes in the force of gravity with an instrument called a *gravimeter*. These minute gravitational changes are due to differences in the density

of the underlying rock; information about rock density provides clues as to the composition of the rock, which helps in locating mineral deposits. See this article at the "Geology for Investors" website.

None of this would work if there were no such thing as gravity, or if Newton's Law of Gravity were incorrect to any substantial degree.